OLIVIER'S THEOREM: *ALL* TRIANGLES ARE EQUILATERAL

Construction: Let ABC be *any* triangle.
Bisect angle A to meet the perpendicular bisector of BC in O.
Draw perpendiculars from O to meet the sides of $\triangle$ABC in P, Q and R.

Proof:
There are three possibilities:
(1) O lies *inside*, (2) *outside* or (3) *on* the triangle.

(1) If O lies inside the triangle (Figure 1):

$\Delta \text{OBR} \equiv \Delta \text{OCR}$ ...... $s, \angle, \text{s}$
$\Rightarrow \text{OB} = \text{OC}$ .... (1)

$\Delta \text{APO} \equiv \Delta \text{AQO}$ ...... $\angle, \angle, \text{s}$
$\Rightarrow \text{OP} = \text{OQ}$ .... (2)
$\Rightarrow \text{AP} = \text{AQ}$ .... (3)

$\Delta \text{OPB} \equiv \Delta \text{OQC}$ ...... $90°$, hyp, s: from (1) and (2)
$\Rightarrow \text{PB} = \text{QC}$ .... (4)

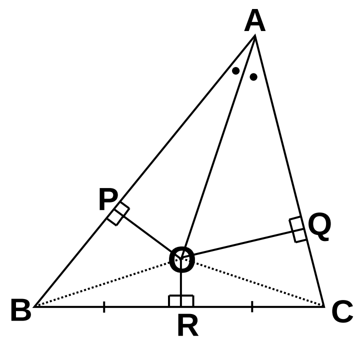


Figure 1

$(3) + (4)$: $\text{AP} + \text{PB} = \text{AQ} + \text{QC}$
$\Rightarrow \text{AB} = \text{AC}$, i.e. ΔABC is isosceles.

(2) If O lies outside the triangle, as in Figure 2:

We prove (1), (2), (3) and (4) exactly as above.

(3) – (4): $\text{AP} - \text{PB} = \text{AQ} - \text{QC}$
$\Rightarrow \text{AB} = \text{AC}$, i.e. ΔABC is isosceles.

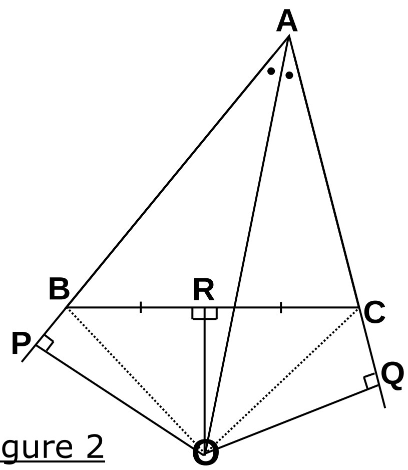


Figure 2

(3) If O lies on BC (Figure 3):

$\text{OB} = \text{OC}$ ...... construction (1)

The rest of the proof is exactly as in Figure 1:

$\text{AP} + \text{PB} = \text{AQ} + \text{QC}$
$\Rightarrow \text{AB} = \text{AC}$, i.e. ΔABC is isosceles.

Thus, *any* triangle is *isosceles*.

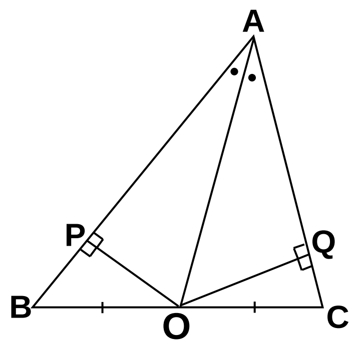


Figure 3

We can similarly prove that in *any* triangle AC = BC.
So, AB = AC = BC, so *any* triangle is *equilateral.*

So *all triangles are equilateral!*

OLIVIER SE STELLING: *ALLE* DRIEHOEKE IS GELYKSYDIG

Konstruksie: Neem *enige* $\triangle$ABC. Halveer hoek A om die middelloodlyn van BC in O te ontmoet. Trek loodlyne vanaf O om die sye van $\triangle$ABC (of hul verlengings) in P, Q en R te ontmoet.

Bewys:
Daar is drie moontlikhede: O <u>lê</u> *binne, buite*, of *op* die driehoek.

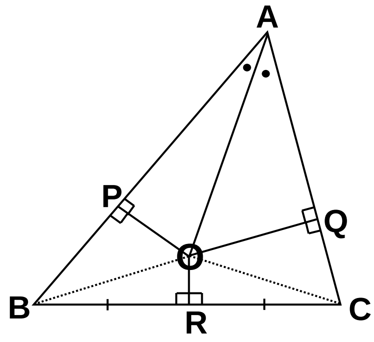
<u>(1) As O binne die driehoek lê, soos in Figuur 1:</u>

$\Delta OBR \equiv \Delta OCR$ ...... $s, \angle, s$
$\Rightarrow OB = OC$ .... (1)

$\Delta APO \equiv \Delta AQO$ ...... $\angle, \angle, s$
$\Rightarrow OP = OQ$ .... (2)
$\Rightarrow AP = AQ$ .... (3)

$\Delta OPB \equiv \Delta OQC$ ...... $90°$, skuins, s: van(1) en(2)
$\Rightarrow PB = QC$ .... (4)

$(3) + (4)$: $AP + PB = AQ + QC$
$\Rightarrow AB = AC$, d.i. ΔABC is gelykbenig.

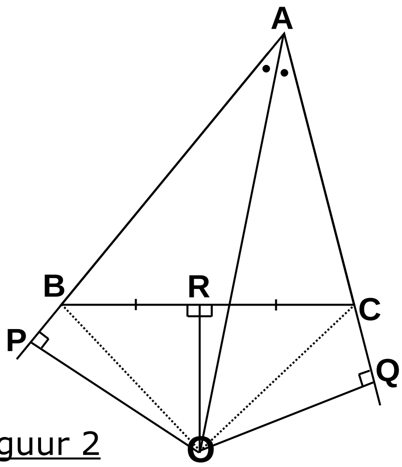


Figuur 1

<u>(2) As O buite die driehoek lê, soos in Figuur 2:</u>

Dan geld (1), (2), (3) en (4) presies soos hierbo.

(3) – (4): $AP - PB = AQ - QC$
$\Rightarrow AB = AC$, d.i. ΔABC is gelykbenig.



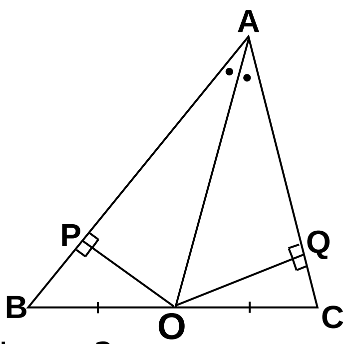
Figuur 2

<u>(3) As O op BC lê, soos in Figuur 3:</u>

$OB = OC$ .... konstruksie (1)

Die res van die bewys volg presies soos in Figuur 1:

$AP + PB = AQ + QC$
$\Rightarrow AB = AC$, d.i. ΔABC is gelykbenig.



Figuur 3

Dus, *enige* driehoek is *gelykbenig*.

Ons kan net so bewys dat in *enige* driehoek is AC = BC.
Dus, AB = AC = BC, dus *enige* driehoek is *gelyksydig*

Dus alle driehoeke is gelyksydig!

ALL TRIANGLES ARE EQUILATERAL – *DISCUSSION*

Of course, *nobody* believes Olivier's Theorem! And *everybody* knows that we can immediately prove that Olivier's Theorem is false by drawing one *counter example …*

But that is not what the activity is about. The educational issue at stake is that logically, if we agree that the *reasoning* in an argument is valid, then we must also accept the *conclusion*, i.e. based on the Modus Ponens proof structure we know

If P ⇒ Q, P is true, *then* Q is necessarily true.

So, if you are not prepared to accept the conclusion that all triangles are equilateral, then the challenge is that you will have to find some mistake in the reasoning. *Where and why is the reasoning invalid?*

So the mathematical activity is to show where there is an error in thinking somewhere in the proof of Olivier's theorem …

If you cannot find an error in thinking, you must logically accept Olivier's theorem, i.e. *you must accept that all triangles are equilateral!*

Prepare a document with your *explanation* to present on TEAMS session in Week 2